Applied Mathematics to Simplify Imager and Camera Analyses

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Abstract. *Discussed in this paper are 3, extremely helpful, techniques in the analyses of imager and camera performance. The first topic is an extension to the well known photon shot noise curve. It is a straight forward method to determine any arbitrary transfer function f(N) from pixel-to-camera output. Next, the design of a μ-lens simulation is usually executed as a function of Chief Ray Angle (CRA). But practically measured with a lens and a given f-number range. The method offered converts the f-numbers into a set of CRA numbers with which the μ-lens efficiency η(CRA) can be determined as a function of CRA. The third topic entails the application of a 2-dimensional histogram to investigate dependence of all pixels in an imager on one parameter. Like temperature, supply voltage or time.*

1.0 An extension to the photon shot noise transfer curve

The noise at the output of a camera consists of two basic parts. One is the system noise often referred to as the read noise and the other is the shot noise. The shot noise is governed by the laws of physics and as such predictable. The system noise can be rather complex depending on whether anomalies in the system occur. Examples of anomalies are: LVDS transmission failures, missing bits in the ADC, periodic ADC noise due to ground bounce. The method and its practical implementation discussed in this paper is applicable in the situation with and without anomalies.
In the linear approach for the shot noise transfer curve the output signal reads

(1.1) $V_{out} = K * N$,

with K the gain from pixel to output and N the number of electrons generated in the pixel. The noise is written as the quadratic sum of the shot noise and the read noise,

(1.2) $U_n = K * \sqrt{N + N_d^{\,2}}$.

For large charge packets (N) the ratio between the noise cubed and the output signal approaches K. Measuring the noise Un as a function of the output signal V_{out} allows to calculate the unknown gain K. Knowing K one can convert the output signals in the number of electrons without a priory knowledge of the μV/e and the gain [1,2] of the camera chain. For instance one can determine the read noise in electrons, the maximum charge handling capacity and other pixel related parameters like FPN and sensitivity all in electrons.
Graphing the noise on the Y-axis and the output signal on the X-axis on a log-log scale shows two asymptotic lines. One for small N values with the noise approaching the read noise N_d and for large N values approaching $\sqrt{N}$ which on a log log scale shows as a straight-line with coefficient ½.

In the general case the output signal V_{out} is written as:

(1.3) $V_{out} = f(N)$

and the corresponding noise, or variance, U_n

(1.4) $U_n = \left|\frac{df(N)}{dN}\right| * \sqrt{N + N_d^{\,2}}$

With f() the transfer function from pixel-(electrons) to-camera-output, N the number of electrons generated in the pixel and N_d the read noise. Under the reasonable assumption of monoticity for f() the absolute-signs can be dropped.
Taking the derivative from equation (1.3) the output signal with respect to the number of electrons N, substituting the result in the noise equation (1.4), applying separation of variables and finally integrating left and right sides, the result reads

(1.5) $\boxed{\int_0^N \frac{dN}{\sqrt{N + N_d^{\,2}}} = \int_0^{Vout} \frac{dV_{out}}{U_n}}$

The left side can be written in closed form and the right side can be measured,

(1.6) $\sqrt{N + N_d^{\,2}} - N_d = \frac{1}{2} * \int_0^{Vout} \frac{dV_{out}}{U_n}$

Hence the arbitrary transfer function f(), from pixel-to-output, can be determined and the output signal V_{out} as a function of the number of electrons (N) generated is known and can be graphed,

(1.7) $\boxed{N = \left(\frac{1}{2} * \int_0^{Vout} \frac{dV_{out}}{U_n} + N_d\right)^2 - N_d^{\,2}}$

Nowadays width the advent of digitized images the method is fairly simple to implement and use is made of the fact that output levels are represented as digital words with a given bit depth. Hence can be used as in index of a one dimensional array too.

The implementation requires 3 grabbed images, Figure 1.1, as follows:
-take **shortly** after each other two snapshots, **ImageG1** and **ImageG2**, of a scene containing all the gray levels from black to white, eg a defocused grey chart. The difference between these two images on a pixel-by-pixel basis is only shot noise and read noise times $\sqrt{2}$.
-take a snapshot of the black, **ImageB**, eg with capped lens. The difference between ImageG1 and ImageB is the output level on a pixel-by-pixel basis. The graph of the noise versus output level, is then calculated following:

For i,j = 1,1 to Nrows, Ncolumns do
Vout= ImageG1[i,j]-ImageB[i,j]);
Histo(Vout)= Histo(Vout)+1;
Variance(Vout) = Variance(Vout) +
+{(imageG2[i,j]-imageG1[i,j])²/2-Variance(Vout) }/Histo(Vout);
End

NOTE: The Variance is written as a moving average recursive form!

For all output levels, $V_{out,}$ now calculate

(1.8) $U_n(V_{out}) = \sqrt{Variance(V_{out})}$

and so the right-hand side of equation (1.6) is known and the pixel-to-output transfer function can be graphed

Figure 1.2 shows the noise as a function of output level for a camera with gamma switched on (power law) and off (linear).

The discretized form of equation (1.7) reads:

(1.9)

$$N_k = \left(\frac{1}{2} \sum_{i=1}^{i=k} \frac{Vout(i) - Vout(i-1)}{U_n(i)} + N_d \right)^2 - N_d^{\ 2}$$

Application of eq. (1.9) to the data depicted in Figure 1.2 results in the second graph, Figure 1.3, where the transfer-function of the pixel-to-camera output is calculated. Clearly the linear and the gamma transfer characteristic are visible.

2.0 Conversion from f-number to chief ray angle

A nice theoretical approach to the optical efficiency is given in [3]. In this chapter a more practical approach is described.

Given a camera, an imager and a lens with aperture f. Than a cone of light with Chief Ray Angles ranging from 0 to CRA_x is projected on the pixels , Figure 2.1, with the following relation between maximum CRA_x and f-number

(2.1) $\boxed{CRA_x = ATAN\left(\frac{1}{2*F}\right)}$

In general the output level V_{out} is proportional with the inverse of the f-number squared, F^{-2}

After applying geometry one arrives at the following relation between output level and CRA_x

(2.2) $\boxed{V_{out} \approx \int_0^{CRA_x} \eta(\theta) * \frac{\tan(\theta)}{\cos(\theta)^2} d\theta}$

Where the μ-lens efficiency, as a function of CRA, is defined as $\eta(CRA)$.

In the case of a perfect μ-lens, $\eta(CRA) = 1$ and after performing the integration of equation (2.2) the output level is proportional with $\tan(CRA_x)^2$ or after substitution of equation (2.1) with the inverse of the f-number squared as one would expect.

Table 1, shows the CRA values for several f-numbers

F	CRA
1.2	22.62
1.4	19.65
2	14.04
2.8	10.12
4	7.13
5.6	5.10
8	3.58
11	2.60
16	1.79

Table 1: CRA expressed in degree

Using the mean-value theorem of integration, defining $\overline{CRA}$ as an element of the interval $[CRA_1, CRA_2]$ eq. (2.2) is than evaluated as

(2.3)

$$V_{out}(CRA_2) - V_{out}(CRA_1) \approx$$

$$\eta(\overline{CRA}) * \int_{CRA_1}^{CRA_{2x}} \frac{\tan(\theta)}{\cos(\theta)^2} d\theta =$$

$$\eta(\overline{CRA}) * \frac{1}{2} * \left(\frac{1}{\cos(CRA_2)^2} - \frac{1}{\cos(CRA_1)^2} \right)$$

Therefore given two f-numbers for which the output level V_{out} is measured, the related CRA can be calculated and an estimation of the μ-lens efficiency on the interval [CRA_1,CRA_2] is determined through equation (2.3)

A more refined approach is by defining the μ-lens efficiency as

$$(2.4)\quad \eta(CRA) = \frac{1}{1 + b * CRA^3} + a * CRA^2 .$$

Substituting into eq. 2.2 and applying a least-mean-square-fit to the measured V_{out}, as a function of CRA_x, the parameters a and b are determined. Hence the *μ-lens* efficiency is known as a function of CRA by substituting back into equation (2.4).

This theory now is applied to experimental results [4] which are graphed for a straight forward μ-lens and a double μ-lens. The μ-lens efficiency as a function of CRA is depicted in figure 2.2. The effect of the embedded lens now clearly shows for CRA larger than 12 degree.

The error between the measured and the estimated V_{out} is within 1% for the new μ-lens and 2% for the old μ-lens.

3.0 On the use of a 2-dimensional-histogram.

The application of a 2-dimensional histogram is in determining activation energies of large population of pixels. Or the dependence on a parameter like pixel supply voltage or change in pixels as a function of time. Its purpose is that after application of the method one can see at a glance if a large amount of pixels on an individual basis behave the same or if the relation to the parameter under investigation is uncorrelated.

The 2-dimensional histogram is an image where X and Y-axis are amplitude values and the Z value is the histogram (amplitude) part.

Use is made of the fact that the pixel amplitude in a digitized image can be used as an index of an array.

The 2-dimensional histogram is generated through amplitude transformation into X or Y position. And intensity as the number of pixels having that joint X,Y amplitude. One needs 2 grabbed images, and only parameter changed in value. Example: **Image1** and **Image2** are 1920x1080x10bit images and temperature was 70C for the first and 60C for the second.

A normal histogram can be generated through

For i:=1 to Nrows

For j:=1 to Ncolumns

HISTO2D[IMAGE1[i,j] ; IMAGE2[i,j]]:= HISTO2D[IMAGE1[i,j] ; IMAGE2[i,j]]+1

This normal histogram shows more or less where and how the point of gravity of all the pixels values changes and are located.

A specialized 2-dimensional histogram is the binarized one [5]:

For i:=1 to Nrows

For j:=1 to Ncolumns

HISTO2DBIN[IMAGE1i,j] ; IMAGE2[i,j]]:=1

If there is a combination of amplitudes that only one pixel exhibits it will show up clearly in the binarized 2-dimensional histogram. As such a very powerful tool to investigate FPN and its excursions, the leaking pixels

Figure 3.1 shows an example of such binarized histogram. It shows that almost all the pixels in the imager have about the same activation energy. With the exception of a few others that go astray.

Figure 3.2 shows an example of a normal 2-dimensional histogram where the parameter changed is the pixel supply voltage. There 3 regions to be discriminated:

1: is where pixels have the same amplitude under both pixel supply voltages, (on the dotted line);

2: where the bulk of the pixels change with the same growth factor as a function of change in pixel supply voltage; (on a angle different from the dotted line)

3: where pixels vanish at low voltage and

4: where pixels vanish at high voltage.

Literature

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[2] J. Mullikin, L. van Vliet, H. Netten, F. Boddeke, G. van der Felz, L, Young, *Methods for CCD Camera Characterization*, SPIE 2173, “Image Acquisition and Scientific Imaging Systems”, 1994, pp 73-84.

[3] P. Catrysse, B. Wandell, *Optical efficiency of image sensor pixels*, J. Opt. Soc. Am. A, Vol. 19, No. 8, August 2002, pp1610-1620.

[4] H. Takahashi et. al., *A 1/2.7’ inch Low-Noise CMOS Image Sensor for Full HD Camcorders* , Visuals Supplement ISSCC 2007, Februari 2007, pp 745

[5] P. Centen et. al., *A 2/3 inch CMOS Image Sensor for HDTV Applications with Multiple High-DR Modes and Flexible Scanning*, Visuals Supplement ISSCC 2007, Februari 2007, pp 421

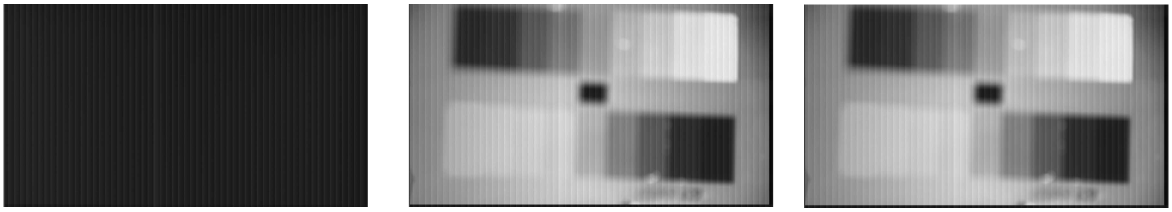

Figure 1.1: Example of a set of 3 images to determine the Shot Noise Transfer Curve

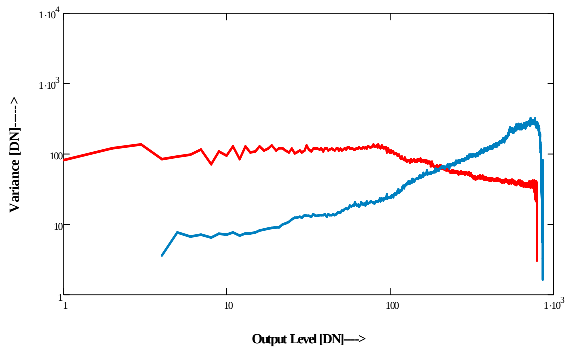

Figure 1.2: Noise (Variance) as a function of output level

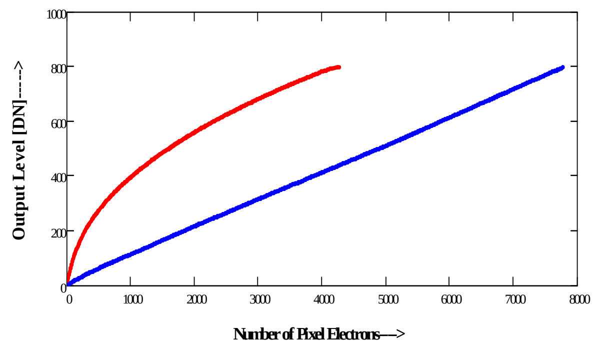

Figure 1.3: Camera Transfer Curve calculated from the shotnoise curves in Figure 1.2.

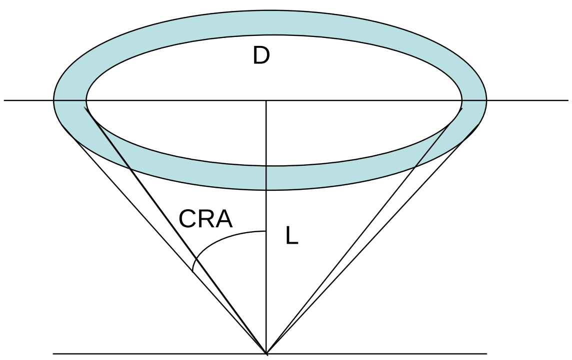

Figure 2.1: F-number and cone angle which is the maximum CRA. All light rays have angles in an interval of [0,CRA].

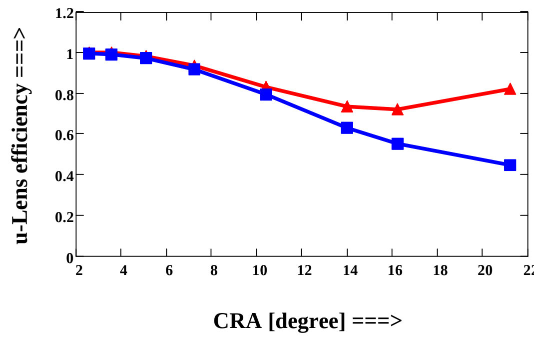

Figure 2.3: Estimated µ-lens efficiency as a function of Chief Ray Angle. Solid squares depict the old µ-lens and solid triangles the new double µ -lens.

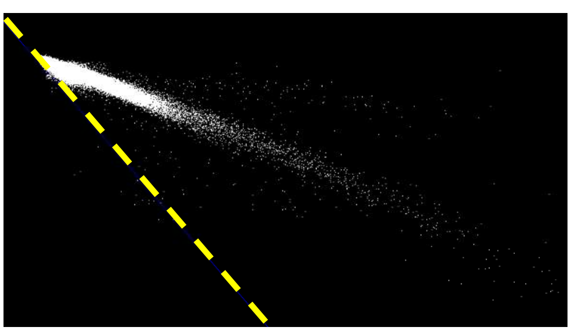

Figure 3.1: Two dimensional binerized histogram for images taken at 70C (X-axis) and 50C (Y-axis). Dotted line for "X=Y"

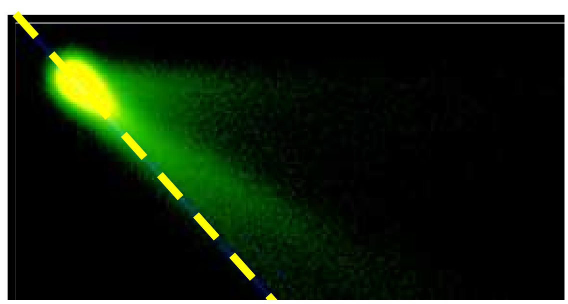

Figure 3.2 A 2-dimensional histogram for an imager with changed pixel voltage. Yellow a linear and green a logarithmic representation. Dotted line for "X=Y".