On the variance and covariance in the determination of PTC

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Agenda

- Shotnoise: deterministic and random events
- PTC in Astronomy, work of Downing et.al. was a trigger
- Correlation between neighboring pixels
- Covariance and Variance

Pre-recorded…. Skipping the powerpoint rules wrt the amount of info on the slide☺

Shotnoise

$$Pois(p,P) = \frac{e^{-P} \cdot P^p}{p!}$$

$$Bin(n,p,\eta) = \frac{p!}{n! \cdot (p-n)!} \cdot \eta^n \cdot (1-\eta)^{p-n}$$

$$\sum_{p=n}^{\infty} Bin(n,p,\eta) \cdot Pois(p,P) = Pois(n,\eta \cdot P) = Pois(n,N)$$

- Photons p: Poisson distribution, with average P and variance P
- photon-to-electron conversion in Si: either it happens, or it doesn't. Binominal with 0<=n<=p and $\eta = QE_{eff}$ It is a random process.
- After cascading it is again Poisson, with avg = N = η*P and var = N = η*P.
- *Note:*
  deterministic e.g. gainstage (K [DN/e])
  *avg = K*N = K*η*P and var = K^2*N = K^2*η*P.*
  The basis for Photon Transfer Curve (PTC).

PTC in Astronomy

Number check
Var=70 S=3000=>K=0.023ADU/e=>43e/ADU
Qlin=65kel, Qx=193kel
4500adu/40sec=112.5adu/sec=>4822e/sec
12um^2=>33e/sec/um^2.....0.7e/um^2 at 20ms
P-type or N-type channel
low and high gain
70um high res

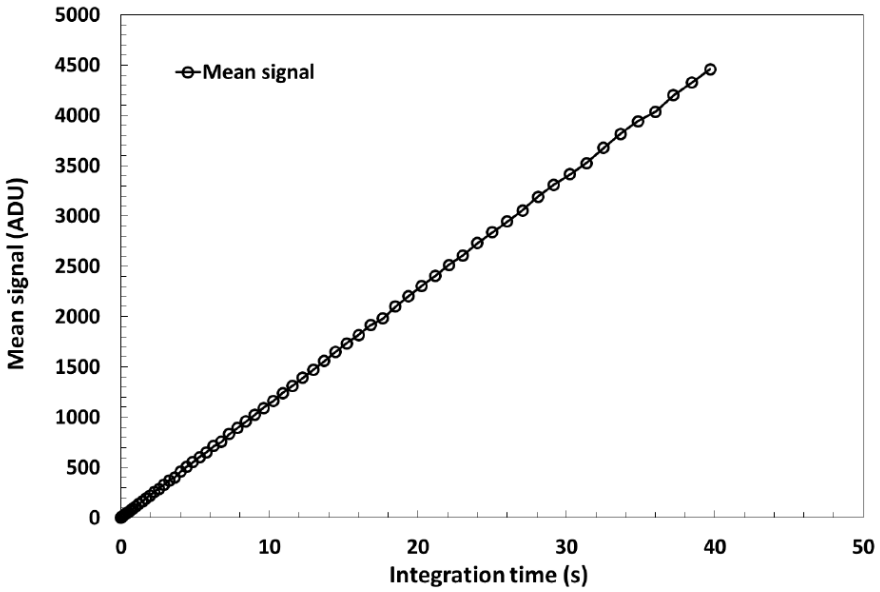

Fig. 2. Photo response of CCD204 obtained with light, with a wavelength of 660 nm. The linearity is better than ±1%.

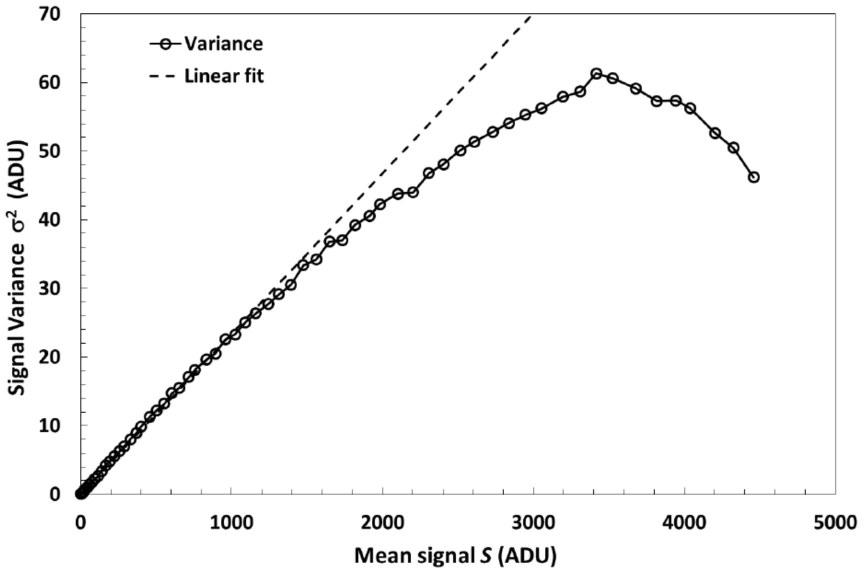

Fig. 1. Photon transfer curve of CCD204 obtained with light, with a wavelength of 660 nm.

Figures from: Konstantin Stefanov, A Statistical Model for Signal-Dependent Charge Sharing in Image Sensors, IEEE January 2014, Transactions on Electron Devices, vol. 61, issue 1, pp. 110-115.

NOTE: Earliest paper: **Mark Downing, et.al.,** *CCD riddle: a) signal vs time: linear; b) signal vs variance: non-linea*r, June 2006, Proceedings of SPIE - The International Society for Optical Engineering 6276**.**

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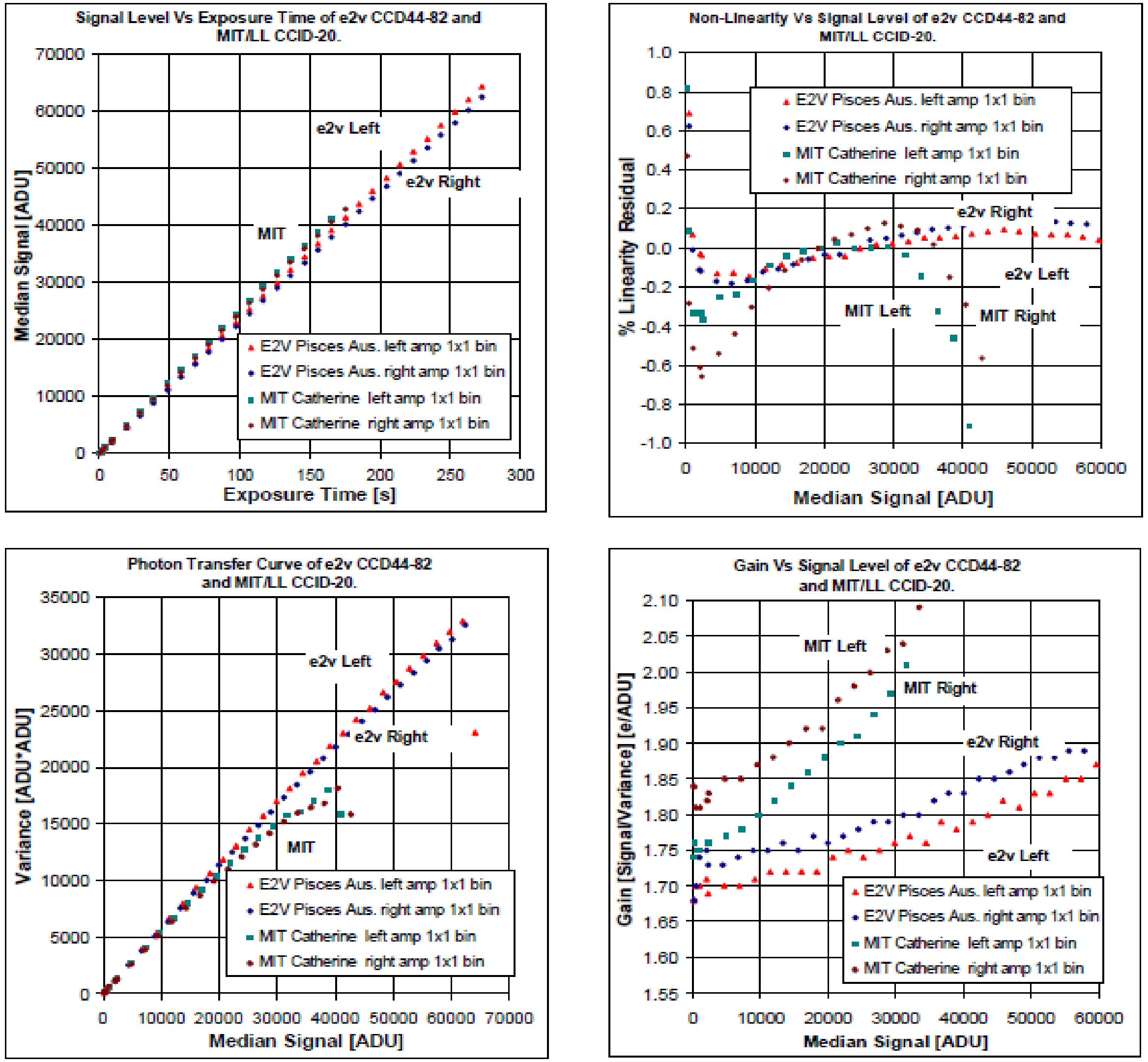

Fig. 2: Signal linearity and photon transfer curves of the left and right amplifiers of MIT/LL Catherine and e2v Pisces Australis II. Top Left: Signal versus exposure time; Top Right: Residual linearity curve; Bottom Left: Photon transfer curve (Variance versus Median Signal); Bottom Right: Calculated gain versus signal.

Figures from:

Mark Downing, et.al., *CCD riddle: a) signal vs time: linear; b) signal vs variance: non-linear*, June 2006, Proceedings of SPIE - The International Society for Optical Engineering 6276**.**

Note:

E2V : CCD44-82; 15um pixels; 16um thick

MIT/LL: CCID-20; 15um pixels; 40um high res thick

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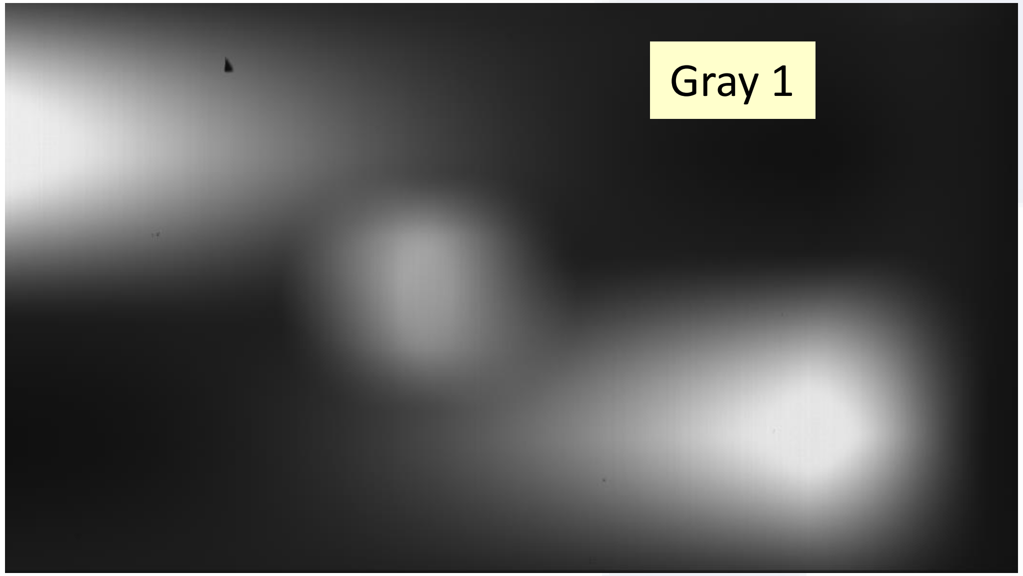


A Low Effort Method to determine PTC

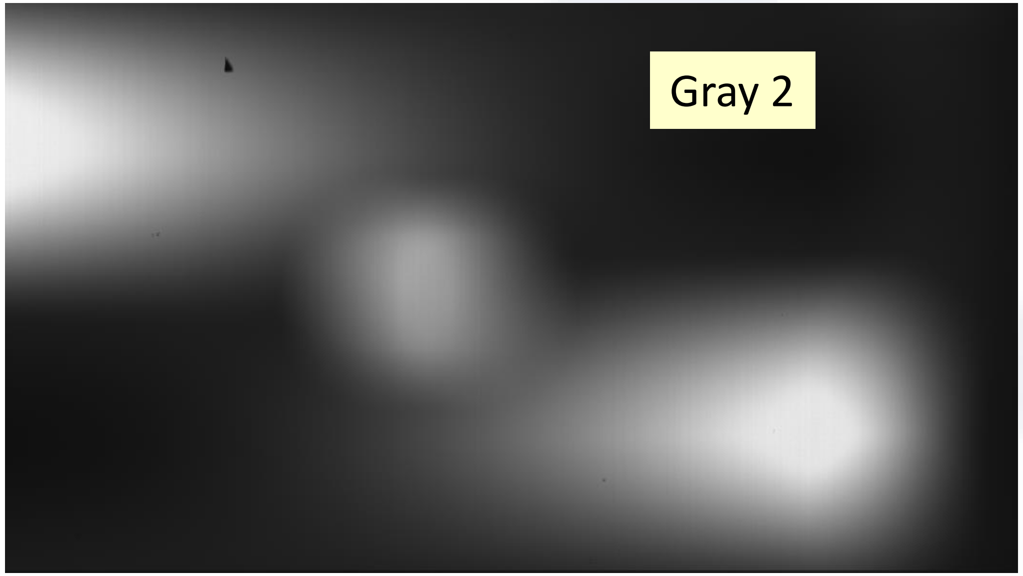
1 image: no illumination, eg capped (Dark)
2 images: same illumination (Gray1 and Gray2) short after one other

Level = (Gray1 + Gray2)/2 – Dark % signal level
Shot = (Gray1 - Gray2)/ $\sqrt{2}$ % includes the readnoise as well

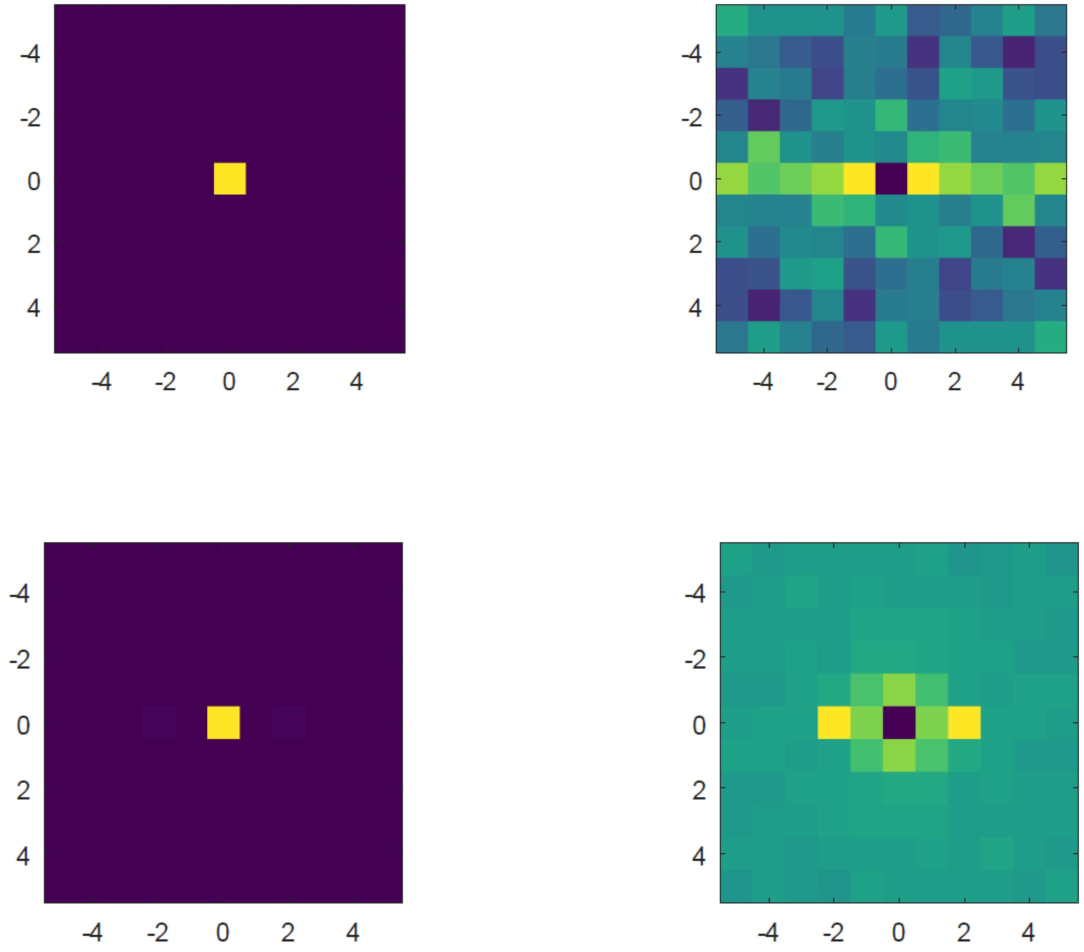
For all pixels (i,j)
HISTO(Level) = HISTO(Level) + 1 ; %Level(i,j) is used as a pointer
VAR(Level) = VAR(Level) + $Shot(Level)^2$; %cumulating the variances
AVG(Level)=AVG(Level)+Shot(Level); %the averages should be close to zero

For i,j,k<>0,l<>0 COVAR(Level(i,j)) = COVAR(Level(i,j)) + Shot(i,j)*Shot(i-k,j-l)

For all Level
VAR(Level)=VAR(Level)/HISTO(Level);
AVG(Level)=AVG(Level)/HISTO(Level);
COVAR(Level)=COVAR(Level)/HISTO(Level);



Correlation with neighboring pixels



- Two different imagers
- The surface plots of the correlation coefficient are autoscaled
  - Left plot: central element (0,0) has value 1
  - Right plot: central element (0,0) made 0 to enhance visibility of the correlation coefficient
- Left plot to show that on a linear scale the correlation is small
- Right upperplot: median=0.14%, max=0.3%
- Right lowerplot: median=2.2%, max=4%

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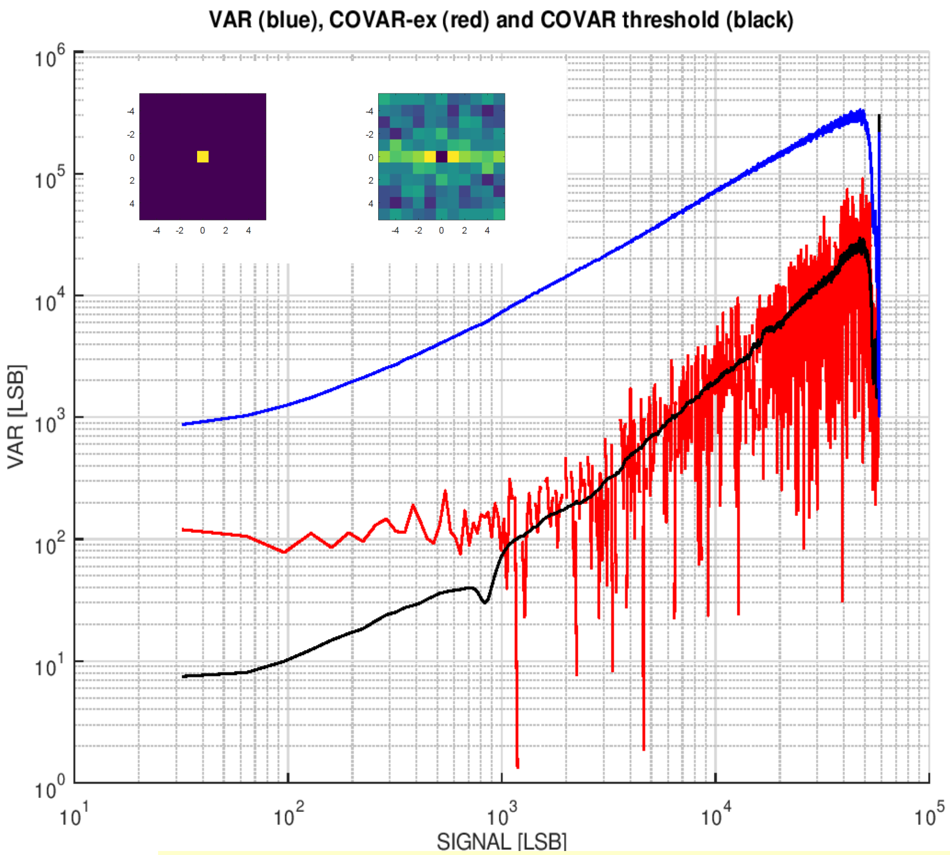
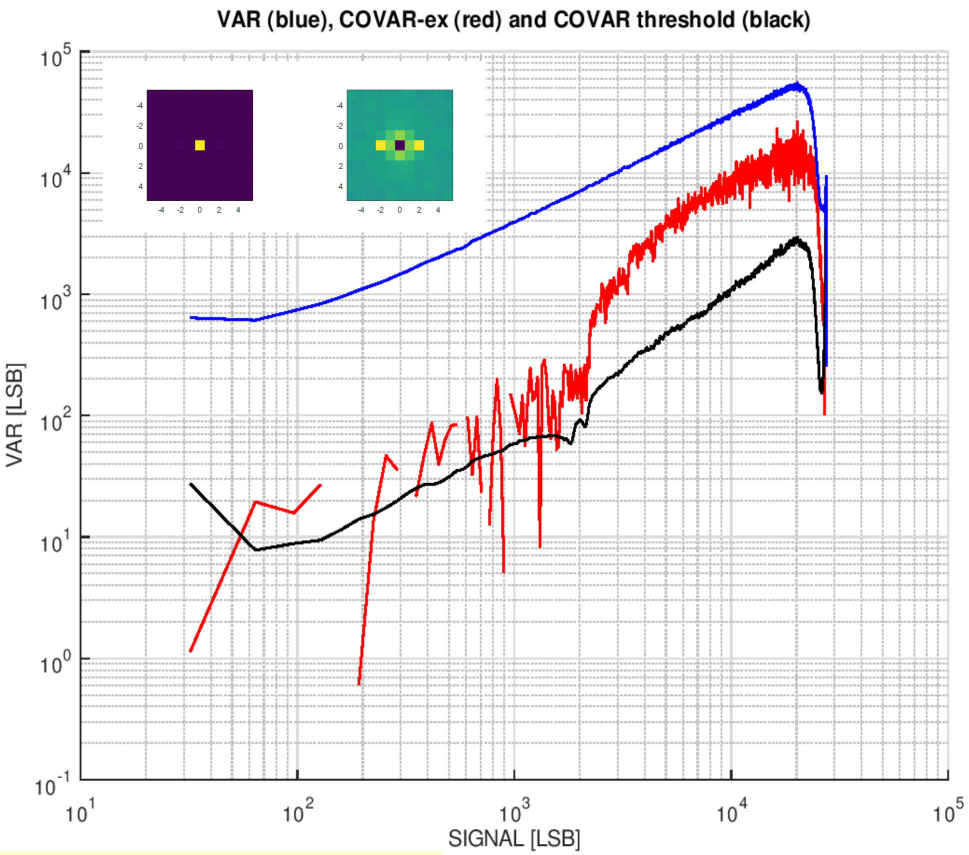
Covariance and variance





Blue trace is variance (PTC),
Red trace is covariance with neighboring pixels in a 3x3 kernel excluding the pixel itself,
Black trace is detection threshold.

Wrap-up

- Check with covariance and/or correlation if there is crosstalk into neighboring pixels. If so, the validity of the Photon Transfer Curve PTC can be in doubt and some investigation is needed.
  - The Photon-Transfer-Curve can be non-linear even when the transfercurve is lineair
  - There is a case with thick BSI CCD's where the covariance can be used to compensate for the non-linearity. Unfortunately, this is not universal applicable.

Slide 6:

E-mail the "Gray1, Gray2 and Dark" in tiff to *peter@peerimaging.com* and I'll discuss the result of your imager/camera in the breakout session. For "Gray" any defocused illumination pattern is ok but no steep edges.